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Admissible Sampling Strategies for Model Sampling from A Finite Population

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Summary

Model sampling is an old practice. To study the relative precision of comparable estimators, it is a usual practice to assume a feasible model namely, population with linear trend, periodic variation, ratio and regression models when auxilliary character is known. In view of the availability of so many models in the literature, it seems interesting to work out unified theory of sampling from a finite population for estimating, as usual, the population total under a general model of the form

$$y_\lambda = a\,\varphi(x_\lambda) + b\,\psi(x_\lambda).$$

Admissible sampling strategies have been presented for linear unbiased estimation of the population total under the above model based on a single observation.

Key words : Model sampling, Auxilliary character, Linear ordering, Admissible strategy, Optimum strategy.

Introduction

MODEL - The problem of model sampling is not a new addition. Ratio method of estimation is optimum when the model is of the form $y_i = R\,x_i$ where (y_i, x_i) denotes values of the character under study and that of an auxilliary variable for the ith population unit. Under the model $y_i = a + bx_i$ difference estimators [2] and regression estimators have been studied. Also to compare various estimators it is an usual practice to assume a feasible model [1].

In the present work, in place of comparing strategies through models, a unified theory of sampling from a finite population of N units is worked out for estimating the population total under a general model

$$y_\lambda = a\,\varphi(x_\lambda) + b\,\psi(x_\lambda), \qquad \lambda = 1, 2, \ldots, N \tag{1}$$

Thus, here a model is assumed and strategies are built up to suit the model.

In case $x_\lambda \propto \lambda \;\forall\; \lambda$, the reduced model becomes

$$y_\lambda = a\,\varphi(\lambda) + b\,\psi(\lambda). \tag{2}$$

Justification for (2) can be given as follows. y_λ regarded as a function of λ, it can always be expressed in a polynomial form of degree (N–1). If in this polynomial representation only two parameters are unknown and different from zero we get (2). Again, a partitioning of λ^i's in the above polynomial representation into two groups such that in each group coefficients are all known except for a common unknown constant may lead to model (2).

Sinha [3] studied the optimum sampling strategies in the class of Linear Unbiased Estimators (LUE) when

$$\varphi(\lambda) = \text{constant}, \ \psi(\lambda) \propto \lambda, \tag{3}$$

i.e.,

$$y_\lambda = a + b\lambda \tag{4}$$

PRESENT WORK — The population total (T) under the model (1) being a function of two unknown parameters a and b we estimate T in terms of a single observed value of y_λ. We present in this process (i) an admissible LUE of T in the restricted class of nonnegative estimators and (ii) minimum variance LUEs of T in some restricted classes of estimators and their admissiblility in the class of LUEs.

The problem discussed here may have direct real life applications in Statistical Quality Control where efficiency of some equipments (e.g. tool wear) steadily declines over time. Indirect application of these results is possible if one considers the sampling unit as a clusture of ultimate units. The observations made here are of theoretical interest too because of the fact that it illustrates many interrelated basic concepts.

2. *Main Results*

THE SET UP — Consider a finite population of N identifiable units $U = (U_1, U_2, \ldots, U_N)$. Let y be the character under study and assume value $y_\lambda = y(U_\lambda)$ on the λth unit, $\lambda = 1, 2, \ldots, N$. Suppose x is an auxilliary character assuming value $x_\lambda = x(U_\lambda)$ on the λth unit, $\lambda = 1, 2, \ldots, N$. As a matter of fact, assume x_λ's to be known before hand. Model under consideration is given at (1) and can be rewritten as

$$y_\lambda = a\,\varphi_\lambda + b\,\psi_\lambda, \tag{5}$$

where $\varphi(x_\lambda) = \varphi_\lambda$, $\psi(x_\lambda) = \psi_\lambda$. Suppose exact forms of Φ_λ and ψ_λ are unknown and our unknown parameters (a, b) constitute a parametric space

$$\textcircled{H} = \{(a,b) \mid 0 \le a, b < \infty\}. \tag{6}$$

As stated earlier the problem is to estimate the population total

$$T = \sum_{\lambda=1}^{N} y_\lambda = a\varphi + b\psi \tag{7}$$

where $\varphi = \sum_{\lambda=1}^{N} \varphi_\lambda$, $\psi = \sum_{\lambda=1}^{N} \psi_\lambda$ say. In sample survey problems, in general, y_λ values are observed to be non-negative so that one may take

$$\psi_\lambda > 0, \ \varphi_\lambda > 0, \quad \forall \quad \lambda, \tag{8}$$

which are sufficient to make $y_\lambda \geq 0 \ \forall \ \lambda$. Because the units are distinguishable and the number of unknown parameters is 2, for samples of size ≥ 2 we can solve for exact values of a and b and hence get the value of T without any error. Thus our problem becomes non-trivial in the case for sample size n = 1 except for the case,

$\varphi : \psi :: \varphi_{\lambda^*} : \psi_{\lambda^*}$ for some $\lambda^* \in (1, 2, \ldots, N)$ as the problem of estimation becomes trivial and we can get the exact value of T even under n = 1.

First of all rearrange the units as $(U_{i_1}, U_{i_2}, \ldots, U_{i_N})$ where $(i_1, i_2, \ldots, i_N)$ is an arrangement of $(1, 2, \ldots, N)$ according to the rule and let us denote it by

$$\frac{\psi_{i_1}}{\varphi_{i_1}} < \frac{\psi_{i_2}}{\varphi_{i_2}} < \ldots < \frac{\psi_{i_N}}{\varphi_{i_N}} \tag{9}$$

and let us denote it by $(U_1, U_2, \ldots, U_N)$ for symbolic simplicity. Confine our attention to the class of LUE of T.

COMPARISON CRITERION — A sampling strategy can be written as

(i) scheme - Draw the i-th unit with probability of selection p_i so that

$$\sum_{i=1}^{N} p_i = 1 \tag{10}$$

(ii) estimator — For the i-th sample $s_i = (U_i)$ we consider

$$e_i = e(s_i, y_i) = c_i \, y_i \tag{11}$$

$i = 1, 2, \ldots, N.$

Different sampling strategies are to be compared through variance of the proposed estimator. In case the variance of the estimator is minimum for all choices of (a, b) in the parametric space (H) we call it the optimum strategy. In the absence of the optimum strategy we adopt admissible strategy that ensures minimum variance of the estimator for at least one choice of (a, b).

BASIC RESULTS — It is difficult to understand how an estimator e makes sense unless $c_i > 0 \ \forall \ i$ in (11). Under such a strategy for e to be unbiased we must have

$$\text{or,} \qquad \Sigma \ c_i \varphi_i p_i = \varphi \tag{12}$$

$$\Sigma \ c_i \psi_i p_i = \psi \tag{13}$$

Thus in the class of LUE, $\mathcal{C}$, the strategy is specified by the following conditions

Scheme : $p[s_i] = p[U_i] = p_i \geq 0 \quad \forall \ i$ where

(i) $\Sigma \ p_i = 1$

with estimator $e(s_i, y_i) = c_i y_i$

$$\begin{aligned} &\text{(ii)} \quad c_i > 0 \ \forall \ i \\ &\text{(iii)} \quad \Sigma \ c_i \varphi_i p_i = \varphi \\ &\text{(iv)} \quad \Sigma \ c_i \psi_i p_i = \psi \end{aligned} \tag{14}$$

Remarks 1. Because the trivial case $\varphi : \psi :: \varphi_\lambda^* : \psi_\lambda^*$ is excluded. Number of nonzero p_i's in (14) must be greater than or equal to 2.

2. In order to satisfy (iii) and (iv) we must have

$$\left.\begin{aligned} &\text{at least one i satisfying } \frac{\psi_i}{\varphi_i} < \frac{\psi}{\varphi} \\ &\text{and at least one i satisfying } \frac{\psi_i}{\varphi_i} > \frac{\psi}{\varphi} \end{aligned}\right\} \tag{15}$$

3. In view of remark 2 and existence of an integer r such that

$$\psi_r/\varphi_r < \psi/\varphi < \psi_{r+1}/\varphi_{r+1} \tag{16}$$

an usable strategy is one where at least one of $(p_1, p_2, \ldots, p_r)$ and $(p_{r+1}, p_{r+2}, \ldots, p_N)$ is non-zero. It is easy to observe that strategies of the following nature are unusable

(i) $p_i = 0$ for $i \geq k, k < r$

(ii) $p_i = 0$ for $i \leq k', k' > r$.

ADMISSIBLE SAMPLING STRATEGY — Consider 2-point strategy defined within the class (14) by the sampling schemes and estimators as

$$\begin{aligned} &\text{Scheme}: p[u_i] = p_i > 0, p[u_j] = p_j > 0, \; p_i + p_j = 1 \\ &\text{Estimator}: e_i = c_i y_i, c_i > 0, e_j = c_j y_j, c_j > 0 \end{aligned} \tag{17}$$

where $i \leq r < r+1 \leq j$ as noted in remark 3.

Then as $E(e) \doteq T$ we have for a 2-point strategy

$$\text{Var}(e) = a^2 \sum_{\lambda} p_\lambda c_\lambda^2 \varphi_\lambda^2 + 2ab \sum_{\lambda} p_\lambda c_\lambda^2 \varphi_\lambda \psi_\lambda + b^2 \sum_{\lambda} p_\lambda c_\lambda^2 \psi_\lambda^2 - T^2,$$

$$\lambda = i, j$$

$$= Q(a, b) - T^2, \text{ say,}$$

where $Q(a, b) = \binom{a}{b}' \sum \binom{a}{b}$ is a quadratic form in a and b with

$$\Sigma = \begin{bmatrix} \Sigma p_\lambda c_\lambda^2 \varphi_\lambda^2, & \Sigma p_\lambda c_\lambda^2 \varphi_\lambda \psi_\lambda \\ \Sigma p_\lambda c_\lambda^2 \varphi_\lambda \psi_\lambda, & \Sigma p_\lambda c_\lambda^2 \psi_\lambda^2 \end{bmatrix} \tag{18}$$

On simplification from the condition of unbiasedness we get

$$c_i = \frac{(\varphi \psi_j - \psi \varphi_j)}{p_i (\varphi_i \psi_j - \psi_i \varphi_j)}, \quad c_j = \frac{(\varphi \psi_i - \psi \varphi_i)}{p_j (\varphi_j \psi_i - \psi_j \varphi_i)}$$

i.e., there is only one unbiased estimator for a 2 point strategy. If we denote by $w_i = \psi_i/\varphi_i - \psi/\varphi$ then

$$|\Sigma| = \frac{\varphi^4}{p_i p_j} \cdot \frac{w_i^2 w_j^2}{(w_i - w_j)^2} \tag{19}$$

where

$$w_1 < w_2 < .. < w_r < 0 < w_{r+1} < .. < w_N. \tag{20}$$

For the purpose of comparison among strategies, consider the variance of the estimator as the comparison criterion under the given condition of unbiasedness. To obtain an admissible 2-point strategy, note lemma 1.

LEMMA 1 — If $|\Sigma(s*)| < |\Sigma(s)|$ for all usable s, then s* is an admissible strategy.

Now to minimise $|\Sigma|$ given in (19) we have clearly the optimum choice of p_i and p_j as

$$p_{i_{opt}} = p_{j_{opt}} = \frac{1}{2}. \tag{21}$$

Also from (20) we get $i_{opt} = r$, and $j_{opt} = r+1$.

Hence follows the following theorem.

Theorem 1. An admissible 2-point strategy within the class (17) for estimating unbiasedly the population total under the model (5) is given by -

$$\begin{aligned} i_{opt} &= r,\ p_{i_{opt}} = \frac{1}{2},\ c_{i_{opt}} = \frac{2(\psi_{r+1}\,\varphi - \psi\,\varphi_{r+1})}{(\psi_{r+1}\,\varphi_r - \psi_r\,\varphi_{r+1})} \\ j_{opt} &= r+1,\ p_{j_{opt}} = \frac{1}{2},\ c_{j_{opt}} = \frac{2(\varphi_r\,\psi - \varphi\,\psi_r)}{(\psi_{r+1}\varphi_r - \psi_r\,\varphi_{r+1})} \end{aligned} \tag{22}$$

LEMMA 2. Every sampling strategy of the form (14) can be *uniformly* improved upon by some 2-point sampling strategy of the form (17), which may be fictitous as well.

Proof — Define $\Sigma_{①}$ as the sum over all i, $i = 1, 2, \ldots, r$ and $\Sigma_{②}$ as the sum over all i, $i = r + 1, r + 2, \ldots, N$.

Consider a 2-point fictitous strategy as defined below

$$p_i* = \Sigma_{①}\, p_i,\ p_j* = \Sigma_{②}\, p_i$$

with the corresponding estimator $e_1* = c_i*\, y_i*, e_j* = c_j*\, y_j*$ where $c_i* > 0, c_j* > 0$,

$$\left.\begin{aligned} \Sigma_{①} p_i\, c_i\, \varphi_i &= p_i*\, c_i*\, \varphi_i*,\ \Sigma_{②}\, p_i\, c_i\, \varphi_i = p_j*\, c_j*\, \varphi_j*, \\ \Sigma_{①} p_i\, c_i\, \psi_i &= p_i*\, c_i*\, \psi_i*,\ \Sigma_{②}\, p_i\, c_i\, \psi_i = p_j* c_j* \psi_j* \end{aligned}\right\} \tag{23}$$

so that

$$\left.\begin{aligned} p_i*\, c_i*\, \varphi_i* + p_j*\, c_j*\, \varphi_j* &= \varphi \\ p_i*\, c_i*\, \psi_i* + p_j*\, c_j*\, \psi_j* &= \psi \end{aligned}\right\} \tag{24}$$

Let,

$$E(e^2) = \binom{a}{b}' \Sigma_N \binom{a}{b} = Q_N(a, b), E(e*^2) = \binom{a}{b}' = Q*_2\ (a, b). \text{ Then}$$

$$\begin{aligned} Q_N(a,b) &= \Sigma\ p_i c_i^2 y_i^2 = \Sigma_{①} p_i c_i^2 y_i^2 + \Sigma_{②} p_i c_i^2 y_i^2 \\ &\geq \frac{(\Sigma_{①} p_i c_i y_i)^2}{\Sigma_{①} p_i} + \frac{(\Sigma_{②} p_i c_i y_i)^2}{\Sigma_{②} p_i} \\ &= p_i^* c_i^{*2} y_i^{*2} + p_j^* c_j^{*2} y_j^{*2} \\ &= Q_2^*(a,b) \end{aligned} \tag{25}$$

Here "=" arises if an only if $c_i \propto \frac{1}{y_i}$ for $i \in \Sigma_{①}$ as well as for $i \in \Sigma_{②}$. But this cannot hold unless the constants of the proportionalities are functions of T which is clearly not possible. This proves the lemma.

Then from the Lemma 2, we have $|\Sigma_N| > |\Sigma_2^*|$ with $|\Sigma_2^*| > |\Sigma_2|$ following Lemma 1, where Σ_2 is based on (22). Thus $|\Sigma_N| > |\Sigma_2|$.

Thus (22) becomes an admissible strategy in $\mathcal{C}$ the entire class (14) of non-negative LUE of T.

Example 1. Suppose $\varphi_\lambda = 1, \psi_\lambda = \lambda$ as considered by Sinha [3]. Then from (22) we have, for even $N, i_{opt} = r = \frac{N}{2}, j_{opt} = r + 1 = \frac{N}{2} + 1$, with $p_r = \frac{1}{2} = p_{r+1}$ and e_i's defined as usual. Clearly we get $e_\lambda \propto y_\lambda$ because $c_{\frac{N}{2}} = c_{\frac{N}{2}+1} = N$, and the estimator proposed at (22) reduces to Sinha's admissible estimator.

Example 2. Suppose $\varphi_\lambda = 1, \psi_\lambda = x_\lambda$ so that $y_\lambda = a + b\,x_\lambda$. Taking x_λ's to be non-negative our model (5) can be applied here. Then from (22) we have $p_r = \frac{1}{2} = p_{r+1}$ with $i_{opt} = r, j_{opt} = r + 1$ where $x_r < \bar{x} < x_{r+1}$.

3. *Some Further Extensions*

EXTENSION — In sec. 2 at (5) to model $y_\lambda = a\,\varphi_\lambda + b\,\psi_\lambda$ has been subjected to restrictions $(a, b) \in Ⓗ$ where $Ⓗ = \{(a,b) \mid 0 \leq a, b < \infty\}$, $\psi_\lambda > 0$, $\varphi_\lambda > 0$, $\forall\ \lambda$ and the class of acceptable strategies for linear unbiased estimation of T was subjected to restrictions $c_i > 0\ \forall\ i$. However, we shall try to generalise the model and the class of acceptable strategies

Model – $y_\lambda = a\,\varphi_\lambda + b\,\psi_\lambda$, where $\varphi_\lambda \neq 0$, $\psi_\lambda \neq 0\ \ \forall\ \lambda, (a, b) \in Ⓗ$

$$= \{(a, b) \mid -\infty < a, b < \infty\} \tag{26}$$

Consider the class of acceptable strategies as defined below :

$$s_i = (u_i),\ p\,[s_i] = p_i \geq 0 \ \ \forall\ i \text{ where}$$

(i) $\Sigma\ p_i = 1$

with estimator $e(s_i, y_i) = c_i\, y_i \ \ \forall\ i$ (27)

(ii) $\Sigma\ c_i\, \varphi_i\, p_i = \varphi$

(iii) $\Sigma\ c_i\, \psi_i\, p_i = \psi$

Remarks 1. For conditions (ii) and (iii) to be satisfied under the restriction $c_i\, \varphi_i > 0\ (<0)\ \ \forall\ i$

$$\left.\begin{array}{l} \text{at least one } i \text{ must satisfy } \dfrac{\psi_i}{\varphi_i} < \dfrac{\psi}{\varphi} \\ \text{and atleast one } i \text{ must satisfy } \dfrac{\psi_i}{\varphi_i} > \dfrac{\psi}{\varphi} \end{array}\right\} \tag{28}$$

2. For conditions (ii) and (iii) to be satisfied under the restriction $c_i\, \psi_i > 0\ (<0)\ \ \forall\ i$

$$\left.\begin{array}{l} \text{at least one } i \text{ must satisfy } \dfrac{\varphi_i}{\psi_i} < \dfrac{\varphi}{\psi} \\ \text{and atleast one } i \text{ must satisfy } \dfrac{\varphi_i}{\psi_i} > \dfrac{\varphi}{\psi} \end{array}\right\} \tag{29}$$

Then we may define, for the chosen strategy (27) of LUE of T, a subclass $\mathcal{C}^{\circ}$ so that

$$\left.\begin{array}{l} p\,(s_i) = p\,(u_i) = p_i \geq 0 \ \ \forall\ i, \quad e\,(s_i, y_i) = c_i\, y_i \ \ \forall\ i \\ \text{with (i)} \quad \Sigma\, p_i = 1, \ \text{(ii)}\ c_i = \dfrac{\varphi}{\varphi_i}, \ \text{(iii)}\ \Sigma\, \psi_i \dfrac{p_i}{\varphi_i} = \dfrac{\psi}{\varphi} \end{array}\right\} \tag{30}$$

and a sub class $\mathcal{C}^*$ so that

$$\left.\begin{array}{l} p\,(s_i) = p\,(u_i) = p_i \geq 0 \ \ \forall\ i, \ e\,(s_i, y_i) = c_i\, y_i \ \ \forall\ i \\ \text{with (i)} \quad \Sigma\ p_i = 1, \ \text{(ii)}\ c_i = \dfrac{\psi}{\psi_i}, \ \text{(iii)}\ \Sigma\ \varphi_i \dfrac{p_i}{\psi_i} = \dfrac{\varphi}{\psi} \end{array}\right\} \tag{31}$$

ABOUT THE CLASSES $\mathcal{C}^{\circ}$ and $\mathcal{C}^*$ — Denote by $\mathcal{C}$ the general class of strategies (27) sub class of which are $\mathcal{C}^o$ and $\mathcal{C}^*$. We have the following important charactarising properties of $\mathcal{C}^o$ and $\mathcal{C}^*$.

Property 1. Necessary and sufficient condition for an estimator $e \in \mathcal{C}$ to satisfy var (e) $\propto a^2$, is that $e \in \mathcal{C}^*$.

Property 2. Necessary and sufficient condition for an estimator $e \in \mathcal{C}$ to satisfy var (e) $\propto b^2$, is that $e \in \mathcal{C}^\circ$.

Property 3. For an estimator $e \in \mathcal{C}^*$, we can not uniformly improve it by any estimator $e \notin \mathcal{C}^*$.

Property 4. For an estimator $e \in \mathcal{C}^o$, we cannot uniformly improve it by any estimator $e \notin \mathcal{C}^\circ$.

Proofs are immediate for property 1 to propery 4.

OPTIMUM SAMPLING STRATEGIES — Within the 2 point strategies of one can find an optimum strategy as follows :

Consider a 2-point strategy $i \le r$, $j \ge r+1$, $p_i > 0$, $p_j > 0$, $p_i + p_j = 1$ with $e_i = \varphi\, y_i/\varphi_i$, $e_j = \varphi\, y_j/\varphi_j$. Then

$$\text{var}(e) = (e_i - T)^2 p_i + (e_j - T)^2 p_j$$
$$= b^2 \varphi^2 \left\{ \left(\frac{\psi_i}{\varphi_i} - \frac{\psi}{\varphi} \right)^2 p_i + \left(\frac{\psi_j}{\varphi_j} - \frac{\psi}{\varphi} \right)^2_{\pi} p_j \right\}$$

where because of unbiasedness $\psi_i\, p_i/\varphi_i + \psi_j\, p_j/\varphi_j = \psi/\varphi$. Now, to minimize the var (e) under the above condition we have for

$\frac{\psi_r}{\varphi_r} < \frac{\psi'}{\varphi} < \frac{\psi_{r+1}}{\varphi_{r+1}}$, $i_{opt} = r$, $j_{opt} = r+1$ with p_r and p_{r+1} satisfying the conditions.

$$p_r + p_{r+1} = , \frac{\psi_r}{\varphi_r} p_r + \frac{\psi_{r+1}}{\varphi_{r+1}} p_{r+1} = \frac{\psi}{\varphi}.$$

Hence we get an optimum strategy in the class of 2-point strategy of $\mathcal{C}^o$ as $\mathcal{C}^\circ$ the probability of selecting u_r is

$$\frac{\dfrac{\psi_{r+1}}{\varphi_{r+1}} - \dfrac{\psi}{\varphi}}{\dfrac{\psi_{r+1}}{\varphi_{r+1}} - \dfrac{\psi_r}{\varphi_r}}$$

$$= \frac{\varphi}{\varphi_{r+1}} y_{r+1} \text{ where } p_{r+1}\text{, the probability of selecting} \quad (32)$$

u_{r+1} is $1 - p_r$.

LEMMA 3 — In the class of strategies $\mathscr{C}^o$, for any given strategy we have a uniformly better 2-point fictitious strategy. The proof follows along the same line of reasons given in Lemma 2 under section 2.

Combining the result (32) and lemma 3 we have the following theorem.

Theorem 2. The 2-point strategy given by (32) is uniformly better than every strategy within the class $\mathscr{C}^o$ for estimating the population total under the model (26)

For $\varphi_s/\psi_s < \varphi < \varphi_{s+1}/\psi_{s+1}$ we can similarly obtain the optimum strategy e* within the class $\mathscr{C}^*$ where

$$\left.\begin{aligned} e^* &= \frac{\psi}{\psi_s} y_s \text{ with } p_s = \frac{(\psi_{s+1}\varphi - \psi\varphi_{s+1})\psi_s}{(\psi_{s+1}\varphi_s - \psi_s\varphi_{s+1})\psi} \text{ as the probability} \\ &\quad \text{of selecting } u_s \\ &= \frac{\psi}{\psi_{s+1}} y_{s+1} \text{ with } p_{s+1} = 1 - p_s \quad \text{as the probability} \end{aligned}\right\} \quad (33)$$

Optimum estimators e^0 and e^* in the subclasses $\mathscr{C}^o$ and $\mathscr{C}^*$ respectively are, in fact, admissible in the general class $\mathscr{C}$ under model (26) for the estimation of the population total and are admissible in the wider class of linear estimators for T iff

$$\frac{\psi}{\varphi} = \frac{1}{2}\left(\frac{\psi_r}{\varphi_r} + \frac{\psi_{r+1}}{\varphi_{r+1}}\right), \quad \frac{\varphi}{\psi} = \frac{1}{2}\left(\frac{\varphi_{s+1}}{\psi_{s+1}} + \frac{\varphi_s}{\psi_{s+1}}\right).$$

Example — Consider $y_\lambda = a + b\,x_\lambda$ where $\varphi_\lambda = 1, \psi_\lambda = x_\lambda$. Then the estimators e^0 and e^* reduce to

(a) $\quad e^0 = Ny_\lambda, \lambda = r \text{ with } p_r = \dfrac{X_{r+1} - \overline{X}}{X_{r+1} - X_r}$ and

$\lambda = r + 1$ with $p_{r+1} = 1 - p_r$ given the condition $X_r < \overline{X} < X_{r+1}$

(b) $\quad e^* = X\dfrac{y_\lambda}{x_\lambda}, \lambda = s \text{ with } p_s = \dfrac{(\overline{X} - X_{s+1})}{(X_s - X_{s+1})\overline{X}}$ and $\lambda = s + 1$

with $p_{s+1} = \dfrac{(X_s - \overline{X})X_{s+1}}{(X_s - X_{s+1})\overline{X}}$ given the condition that

$$\frac{1}{X_s} < \frac{1}{\bar{X}} < \frac{1}{X_{s+1}}, \text{ with } u_i\text{'s arranged in order of} \left\{ \frac{1}{X_i} \right\}.$$

It is obvious that e^0 is the estimator with probability depending on size type, and e^* is the ratio type estimator.

4. *Comments*

Admissible and optimum sampling strategies considered in section 2 and section 3 involve problems related to nonlinear programming. In our statistical treatment we could, however, reduce the dimension of the problem in view of the lemma 2 and lemma 3.

The model examined here is of deterministic nature. An extension of the same in the stochastic domain is under study. Analytical approach being different separate communication will be made on the same for the set up

$$y_\lambda = a\,\varphi(x_\lambda) + b\,\psi(x_\lambda) + e_\lambda$$

where $E(e_\lambda) = 0$, $\text{Var}(e_\lambda) = \sigma^2$ for each λ and the usual assumption of independence of error terms holds.

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